Social Networks and Marriage Matching

Debra Hevenstone

Outline

- Background
  - Matching
  - Dating
  - Hypotheses

- Method
  - Simulation Design
  - Experiment
  - Results

- Future work
  - Simulation
  - Empirical
  - Limitations
Matching Problems

- Goal is a stable match
  - One:one, one:N
  - Bipartite 1:1 (called “Marriage Problem”)
- Algorithms assume
  - Full scope search
  - Lengthy search
- But real marriage search
  - Limited length search
  - Limited scope search
Matching Problems: Algorithms with full scope search

Medical graduates ↔ Hospitals
H.S. students ↔ Schools
Organ donors ↔ Recipients
Rabbinical graduates ↔ Synagogues
Law school graduates ↔ Firms
Matching Problems: Gale-Shapely (G-S) algorithm

- N men and N women rank potential partners (no ties)
- Men propose to the most-preferred woman not yet proposed to
- Women accept if unmatched, or if proposal is improvement
- Up to $n^2 - n + 1$ rounds to stable solution

Stable Solutions

- (AZ, BX, CY)
- (AY, BZ, CX)
- (AX, BY, CZ)
Matching Problems: G-S results & limitations

- **Stable Solution**
  - If a man prefers another, he must have already proposed and been rejected
  - If a woman prefers another, she must be less preferred than his partner

- **Optimality**
  - Men get best feasible partner
  - Women get worst feasible partner

- **Real “marriage problem”**
  - Limited time to search
  - Limited scope for search
Empirical Evidence: Full scope search

- Full scope: Online dating platform
- Limited scope: Personal social network
- G-S algorithm predicting exchange of email/contact info

Internet dating less homophilous than real life (e.g. edu)
Empirical Evidence: Scope limited by dating history

- Simulate relationship histories
- Accurate prediction only limiting scope by relationship history

(Moody, Bearman, Stovel, 2004)
## Hypotheses

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Simulation/Mechanism</th>
<th>Empirical Confirmation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Match Utility</strong></td>
<td>Unable to offer good matches</td>
<td>Less happy relationships meeting through friends</td>
</tr>
<tr>
<td>lower w/ limited scope</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Women’s Utility</strong></td>
<td>Offers to lower-ranked accepted offers</td>
<td>Women happier meeting through friends, less meeting young</td>
</tr>
<tr>
<td>higher w/ limited scope</td>
<td>improve with time</td>
<td></td>
</tr>
<tr>
<td>lower w/ limited time</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dating History</strong></td>
<td>Initial matches follow social network,</td>
<td>Add Health evidence, high school context</td>
</tr>
<tr>
<td>loops with longer time/</td>
<td>then cycle w/in clique</td>
<td></td>
</tr>
<tr>
<td>limited scope, trees with</td>
<td></td>
<td></td>
</tr>
<tr>
<td>limited time/limited scope</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Method: Agent Based Model (ABM)

- Initialize
  - Agents w/ heterogenous characteristics & preferences
  - Calculate match utility & partner rankings
- Construct social network
  - Set parameters
  - Assign edges
- Run G-S algorithm
  - Full scope (varied search length)
  - Scope limited to 2-hop network (varied search length)
Method: Initialization, heterogeneous agents

- N male agents & N female agents
- Symbolic characteristics & preferences

<table>
<thead>
<tr>
<th>Attractiveness</th>
<th>( a = \text{norm}(\mu = 5, \sigma = 1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intelligence</td>
<td>( s = \text{norm}(\mu = 5, \sigma = 1) )</td>
</tr>
<tr>
<td>Preference for partner's ( a )</td>
<td>( \alpha = \text{uniform}(0, 1) )</td>
</tr>
<tr>
<td>Preference for partner's ( s )</td>
<td>( = (1 - \alpha) )</td>
</tr>
</tbody>
</table>
Method: Initialization, match utility & partner rankings

- Cobb Douglas match utility
- Constant returns to scale
- Function of partner’s characteristics & ego’s preferences

\[ u_i = a_j^{\alpha_i} s_j^{1-\alpha_i} \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_i )</td>
<td>Utility of ego ( i )</td>
</tr>
<tr>
<td>( a_j )</td>
<td>Attractiveness of partner ( j )</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Importance of attractiveness in partner for ego ( i )</td>
</tr>
<tr>
<td>( s_j )</td>
<td>Intelligence of partner ( j )</td>
</tr>
<tr>
<td>( 1 - \alpha_i )</td>
<td>Importance of intelligence in partner for ego ( i )</td>
</tr>
</tbody>
</table>
Method: Construct network, parameter settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dating Offers</td>
<td>1</td>
</tr>
<tr>
<td>Number of Networks</td>
<td>1</td>
</tr>
<tr>
<td>Average Network Degree (should be &lt;(n-1)/2)</td>
<td>3</td>
</tr>
<tr>
<td>Number of Men</td>
<td>10</td>
</tr>
<tr>
<td>Number of Women</td>
<td>10</td>
</tr>
<tr>
<td>Coefficient: Impact of attractiveness on making new friends</td>
<td>.03</td>
</tr>
<tr>
<td>Coefficient: Impact of intelligence on making new friends</td>
<td>.03</td>
</tr>
<tr>
<td>Coefficient: Impact of similar attractiveness on making new friends</td>
<td>-.1</td>
</tr>
<tr>
<td>Coefficient: Impact of similar intelligence on making new friends</td>
<td>-.1</td>
</tr>
<tr>
<td>Coefficient: Impact of common friends on becoming friends</td>
<td>.1</td>
</tr>
<tr>
<td>Coefficient: Impact of already having friends on becoming friends</td>
<td>.1</td>
</tr>
</tbody>
</table>

Path to where you want to save results: /Users/debraheavenstone/Documents/workspace/
Method: Construct network, assign edges

- Pull two random agents
- Calculate edge probability

\[ Pr_{i,j=1} = \frac{e^f}{1+e^f} \]

\[ f = \beta(x_i + x_j) + \gamma x_{ij} + \psi(D_i + D_j) + \gamma T_{ij} \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i )</td>
<td>individual characteristics (not preferences!)</td>
</tr>
<tr>
<td>( x_{ij} )</td>
<td>similarity of characteristics</td>
</tr>
<tr>
<td>( D_i )</td>
<td>current degree</td>
</tr>
<tr>
<td>( T_{ij} )</td>
<td>number of friends in common</td>
</tr>
<tr>
<td>( \beta, \gamma, \psi, \gamma )</td>
<td>coefficients</td>
</tr>
</tbody>
</table>

- if (random uniform \((0,1) > Pr_{i,j=1}\), \((i \cdot j) = 1\)
Method: Run G-S Algorithm (full & limited scope)

- Agents rank *visible* potential partners (no ties)
- Men propose to the most-preferred *visible* woman (not yet proposed to)
- Women accept if unmatched, or if better proposal
- $X$ offer rounds

Full Scope

Limited Scope (1-hop)
Experiment: G-S matching w/ limited time and scope

- Hypotheses
  - Match Utility
    - Lower w/ limited scope (unable to find best matches)
  - Female Utility
    - Higher with limited scope (easier competition)
    - Lower with limited time (no chance to dump less preferred)
Experiment: Settings

- Constant conditions
  - 50 men and women
  - Agent characteristic distributions ($\text{norm}(\mu = 5, \sigma = 1)$)
  - Coefficients controlling friendship network generation

- Experimental conditions
  - Number of offers: 1:10
  - Full scope search vs limited scope (2-hop) search

- 10 runs per experimental condition
Experiment: Measured output

- **Agent data**
  - Individual characteristics
  - Partner characteristics
  - **Match utility**

- Romantic history network (Intermediate steps)
  - Density at maximum reach
  - Size of the largest component
  - Centralization
    \[
    \frac{\sum (\text{centrality}_i - \text{centrality}_*)}{\max \sum (\text{centrality}_i - \text{centrality}_*)}
    \]
  - Mean geodesic length (avg shortest path)
  - Number of 4-cycles
Experiment: Results, match utility
## Experiment Results, match utility (OLS)

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Effect Type</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ego variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isFemale</td>
<td>-3.68***</td>
<td>built-in</td>
</tr>
<tr>
<td>Attractive</td>
<td>1.00***</td>
<td>built-in</td>
</tr>
<tr>
<td>Intelligence</td>
<td>1.06***</td>
<td>built-in</td>
</tr>
<tr>
<td><strong>Experimental variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of offers</td>
<td>2.43***</td>
<td>built-in</td>
</tr>
<tr>
<td>Number of offers^2</td>
<td>-0.37***</td>
<td>built-in</td>
</tr>
<tr>
<td>isNetworkMatch</td>
<td>5.55***</td>
<td>emergent?</td>
</tr>
<tr>
<td><strong>Experiment-ego interactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isFemale * isNetworkMatch</td>
<td>0.46***</td>
<td>built-in</td>
</tr>
<tr>
<td>Attractive * isNetworkMatch</td>
<td>-0.40***</td>
<td>built-in</td>
</tr>
<tr>
<td>Intelligence * isNetworkMatch</td>
<td>-0.58***</td>
<td>built-in</td>
</tr>
<tr>
<td>isFemale * N. offers</td>
<td>0.46***</td>
<td>built-in</td>
</tr>
<tr>
<td><strong>Ego-ego interactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isFemale * Attractive</td>
<td>0.38***</td>
<td>emergent?</td>
</tr>
<tr>
<td>isFemale * Intelligence</td>
<td>0.51***</td>
<td>emergent?</td>
</tr>
</tbody>
</table>

Also controlling for N. offers * network match
Future Simulation Output: Relationship histories

- Treat intermediate matching steps as relationship history
- Causal mechanism
  
  limited search scope & limited time
  vs
  taboos
Future Simulation: Continuous information limits

- least time
- 1-hop dominates
- middle time
- 2-hop dominates
- unlimited time
- unlimited scope
- simulation length

avg match utility

universal search

search 2-hop

search 1-hop
Future Empirical Work: Erasmus/high school dating & friendship networks

- Limitations
  - Search time 6 mo to 2 yrs
  - Unlimited search has boundary problems per definition
  - Cross sectional & retrospective

- Questions
  - What predicts friendship?
  - Does friendship predict partner?
  - Does partner satisfaction vary if matched over network?
  - Male/female differences?
Empirical Limitations: Timing and causality

- Study using Facebook data
- Goal: Identify the relationship edge
- Two hypotheses
  - Embeddedness
    Number of i & j’s friends in common in i’s network
  - Dispersion
    Dyads w/ no mutual friend once i and j are removed
- Causality
  Dispersion is observed because edges formed post-relationship

Backstrom and Kleinberg, 2013
Conclusion/Who Cares

- Dating/online dating platforms
- Epidemiological implications
- Applications to job search
- Could imposed networks reduce search time?